# DOCTORAL FORUM OF MATHEMATICS BETWEEN FUDAN AND KYOTO UNIVERSITIES

Date: December 8-12, 2012

Place: Faculty of Science Bldg. No. 3 Room 110

#### INVITED SPEAKERS

Shu Kawaguchi (Kyoto University) Zhi Lu (Fudan University)

Jiangang Ying (Fudan University) Mitsuhiro Shishikura (Kyoto University)

Hisashi Okamoto (RIMS) Yongqian Zhang (Fudan University) Engui Fan (Fudan University) Koji Fujiwara (Kyoto University)

	8th (Sat.)	9th (Sun.)	10th (Mon.)	11th (Tue.)	12th (Wed.)
9:30 - 10:30	Shu Kawaguchi	Shuxia Wang	Jiangang Ying	Hisashi Okamoto	Engui Fan
		Ke Wang			
11:00 - 12:00	Zhi Lu	Kanae Akaiwa	Mitsuhiro Shishikura	Yongqian Zhang	Koji Fujiwara
13:30 - 14:00	Mamoru Okamoto	Weimin Peng		Yihang Hao	Yasuhiro Ishitsuka
14:10 - 14:40	Long Hu	Kenta Ishimoto		Kenta Tottori	Jinsong Xu
15:00 - 15:30	Masaki Wada	Caixuan Ren		Pengpeng Xie	Kazuto Ota
15:40 - 16:10	Jiangyan Pu	Miho Hatanaka		Takayuki Okuda	Zhenguo Feng
16:30 - 17:00	Naova Umezaki	Fei Yang		Chong Zhao	Takamori Kato

### November 8. Sat.

<b>A.M.</b> 9:30 - 10:30	Chair: Professors Shu Kawaguchi (Kyoto University) Degrees of iterations of triangular automorphisms over Q-algebras				
11:00 - 12:00	Zhi Lu (Fudan University) Moment-angle complexes and Halperin-Carlsson conjecture				
<b>P.M.</b> 13:30 - 14:00  14:10 - 14:40	Chair: Students Mamoru Okamoto (Kyoto University) Well-posedness for the Chern-Simons-Dirac system in two dimensions Long Hu (Fudan University)				
15:00 - 15:30	Exact boundary controllability and synchronization for a coupled system of 1-D Wave equations  Masaki Wada (Tohoku University)  Perturbation of Dirichlet forms and stability of fundamental solutions				
15:40 - 16:10	Jiangyan Pu (Fudan University) Robust consumption portfolio optimization with recursive utility				
16:30 - 17:00	Naoya Umezaki (University of Tokyo) On the Grothendieck's monodromy theorem				
November 9. Sun.					
<b>A.M.</b> 9:30 - 10:00  10:10 - 10:40  11:00 - 11:30	Chair: Students Shuxia Wang (Peking University) Small energy scattering for the Klein-Gordon-Zakharov system Ke Wang (Fudan University) Boundary feedback stabilization for isothermal Euler equations with friction Kanae Akaiwa (Kyoto University)				
P.M. 13:30 - 14:00 14:10 - 14:40 15:00 - 15:30	On a matrix tridiagonalization technique based on the quotient-difference formula Chair: Students Weimin Peng (Fudan University) Global solutions to incompressible Navier-Stokes equations with gravity Kenta Ishimoto (RIMS) Stokes dynamics of swimming microorganisms Caixuan Ren (Fudan University) Regularization by projection for a backward problem of the time-fractional diffusion equation				

15:40 - 16:10 Miho Hatanaka (Osaka City University)

The uniqueness of decompositions of a (quasi)toric manifold into products of real 2 or 4 dimensional (quasi)toric manifolds

16:30 - 17:00 Fei Yang (Fudan University)

Rational maps whose Julia sets are Cantor circles

#### November 10. Mon.

A.M. Chair: Professors
9:30 - 10:30 Jiangang Ying (Fudan University)
On Kac's scattering length formula
11:00 - 12:00 Mitsuhiro Shishikura (Kyoto University)
Renormalization in complex dynamics

#### November 11. Tue.

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$\mathbf{A}.\mathbf{M}.$	Chair: Professors			
9:30 - 10:30	Hisashi Okamoto (RIMS)			
	Water-waves and drift of particles			
11:00 - 12:00	Yongqian Zhang (Fudan University)			
	Global solution for 2-D steady supersonic flow over a bending wall			
P.M.	Chair: Students			
13:30 - 14:00	Yihang Hao (Fudan University)			
	The existence and blow-up criterion of liquid crystals system in critical			
	Besov space			
14:10 - 14:40	Kenta Tottori (Tohoku University)			
	A Monge-Ampère foliation associated with a geodesic in the space of			
	Kähler metrics			
15:00 - 15:30	Pengpeng Xie (Fudan University)			
	Sharp perturbation bounds for the total least squares problem			
15:40 - 16:10	Takayuki Okuda (University of Tokyo)			
	Classification of semisimple symmetric spaces with properly discontinuous			
	actions of surface groups			
16:30 - 17:00	Chong Zhao (Fudan University)			
	p-essential normality of quasi-homogeneous Arveson's submodules			

### November 12. Wed

A.M.	Chair: Professors		
9:30 - 10:30	Engui Fan (Fudan University)		
	Super extension of Bell polynomials with applications to superymmetric equations		
11:00 - 12:00	Koji Fujiwara (Kyoto University)		
	Quasi-homomorphisms		
P.M.	Chair: Students		
13:30 - 14:00	Yasuhiro Ishitsuka (Kyoto University)		
	Complete intersections of two quadrics and Galois cohomology		
14:10 - 14:40	Jinsong Xu (Fudan University)		
	A characterization of birationality of the fourth pluricanonical map		
15:00 - 15:30	Kazuto Ota (Tohoku University)		
	A generalization of the theory of Coleman power series		
15:40 - 16:10	Zhenguo Feng (Fudan University)		
	Tricomi problem for a quasilinear Lavrentiev-Bitsadze equation of mixed type		
16:30 - 17:00	Takamori Kato (Kyoto University)		
	Well-posedness of the fifth order KdV equation with periodic boundary condition		

#### Degrees of iterations of triangular automorphisms over Q-algebras

Shu Kawaguchi, Kyoto University

**Abstract:** For a birational map f on projective space defined over  $\mathbb{C}$ , behavior of the degrees of the iterations of f (forwardly and backwardly) has been quite studied. In this talk, we would like to study such behavior when  $\mathbb{C}$  is replaced by a  $\mathbb{Q}$ -algebra in general. We consider a simple case, where f is an affine space triangular automorphism defined over a  $\mathbb{Q}$ -algebra.

#### Moment-angle complexes and Halperin-Carlsson conjecture

Zhi Lu, Fudan University

Abstract: We give an algebra-combinatorics formula of the Möbius transform for an abstract simplicial complex K on  $[m] = \{1, ..., m\}$  in terms of the Betti numbers of the Stanley-Reisner face ring of K. Furthermore, we employ a way of compressing K to estimate the lower bound of the sum of those Betti numbers by using this formula. As an application, associating with the moment-angle complex  $\mathcal{Z}_K$  (resp. real moment-angle complex  $\mathbb{R}\mathcal{Z}_K$ ) of K, we show that the Halperin-Carlsson conjecture holds for  $\mathcal{Z}_K$  (resp.  $\mathbb{R}\mathcal{Z}_K$ ) under the restriction of the natural  $T^m$ -action on  $\mathcal{Z}_K$  (resp.  $(\mathbb{Z}_2)^m$ -action on  $\mathbb{R}\mathcal{Z}_K$ ). (Joint work with Xiangyu Cao)

#### Well-posedness for the Chern-Simons-Dirac system in two dimensions

Mamoru Okamoto, Kyoto University

**Abstract:** In this talk, we consider the Cauchy problem for the Chern-Simons-Dirac system in  $\mathbb{R}^{1+2}$ . Using the gauge invariance, we reduce the Chern-Simons-Dirac system to a Dirac equation. We discover the null structure of this Dirac equation. The null structure plays a crucial role in the proof because it improves the product estimates for wave Sobolev spaces. Using the null structure estimates, we show that the Cauchy problem of this Dirac equation is local in time well-posed in  $H^s$  with s > 1/4. For the proof, we employ the Fourier restriction norm methods.

### Exact boundary controllability and synchronization for a coupled system of 1-D wave equations

Long Hu, Fudan University

**Abstract:** In this talk, we firstly present a unified constructive method to establish the exact boundary controllability for a kind of coupled system of 1-D wave equations with Dirichlet type, Neumann type and Coupled third type boundary conditions in the framework of semi-global classical solutions. Based on this, we then introduce exact synchronization for a coupled system of 1-D wave equations with boundary conditions of various types and we show that the synchronization

can be realized by means of less boundary controls. The latter part of this lecture is a recent joint work with Prof. Tatsien Li and Prof. Bopeng Rao.

#### Perturbation of Dirichlet forms and stability of fundamental solutions

Masaki Wada, Tohoku University

**Abstract:** Let  $(\mathcal{E}, \mathcal{F})$  be a symmetric regular Dirichlet form as follows:

$$\mathcal{E}(u,v) = \iint_{\mathbb{R}^d \times \mathbb{R}^d} (u(y) - u(x))(v(y) - v(x))J(x,y)dxdy$$

$$\frac{\kappa_1}{|x - y|^d \phi(|x - y|)} \le J(x,y) \le \frac{\kappa_2}{|x - y|^d \phi(|x - y|)} \quad (0 < \kappa_1 \le \kappa_2)$$

$$\phi(r) = r^\alpha \exp(m(r - 1) \lor 0) \quad (0 < \alpha < 2, m \ge 0)$$

It is proved that the associated fundamental solution p(t, x, y) admits the two-sided estimates by Chen, Kumagai et al. Let  $\mu$  be a positive Radon smooth measure in a certain class and consider the perturbed form  $\mathcal{E}^{\mu}(u,u) = \mathcal{E}(u,u) - \int_{\mathbb{R}^d} u^2(x) d\mu$ . Denote the fundamental solution associated with  $\mathcal{E}^{\mu}$  by  $p^{\mu}(t,x,y)$ . We call it the stability of fundamental solution that  $p^{\mu}(t,x,y)$  has the same two-sided estimates as p(t,x,y) up to positive constants. In this talk, we establish a necessary and sufficient condition on  $\mu$  for the stability.

#### Robust consumption portfolio optimization with recursive utility

Jiangyan Pu, Fudan University

**Abstract:** This talk examines a continuous time intertemporal consumption and portfolio choice problem for an investor with recursive preferences. The investor worries about model misspecification and seeks robust decision rules. We formulate the HJB equation and provide explicit solutions. We also compare our robust solutions to that with non-robustness.

#### On the Grothendieck's monodromy theorem

Naoya Umezaki, University of Tokyo

**Abstract:** Grothendieck showed that the action of Galois group on the etale cohomology, which we call monodromy, can be unipotent by a finite base extension.

In this talk, we introduce the monodromy in above sense and give my result, which says that the degree of base extension can be controlled only by certain invariants of the varieties.

#### Small energy scattering for the Klein-Gordon-Zakharov system

Shuxia Wang, Peking University

**Abstract:** In this talk, I will introduce the radial-improved Strichartz estimates for Klein-Gordon and wave equations; and then prove the small energy scattering of Klein-Gordon-Zakharov system in Sobolev spaces by using normal form reduction and the Strichartz estimates given above.

#### Boundary feedback stabilization for isothermal Euler equations with friction

Ke Wang, Fudan University

**Abstract:** A second-order quasilinear hyperbolic equation is derived that is equivalent to the isothermal Euler equations with friction. For the corresponding mixed initial-boundary value problem, we consider non-stationary solutions locally around a stationary state on a finite time interval and discuss the well-posedness of this kind of problem. We introduce a strict  $H^2$ -Lyapunov function and show that the boundary feedback constant can be chosen such that the  $H^2$ -Lyapunov function and hence also the  $H^2$ -norm of the difference between the non-stationary and the stationary state decays exponentially with time. An exponential estimate for the  $C^1$ -norm is also established. Then, we generalize this result to the gas flows in a planar tree-like network.

#### On a matrix tridiagonalization technique based on the quotient-difference formula

Kanae Akaiwa, Kyoto University

**Abstract:** The quotient-difference (qd) algorithm was proposed by H. Rutishauser in 1954 and is usually known as be applicable to computing matrix eigenvalues with a tridiagonalization technique by the Householder transformation. In this talk, we mainly describe that dense matrix can be reduced into tridiagonal one through using the recursion formula of the qd algorithm instead of the Householder transformation. We also prove that the reduced matrix has the same eigenvalues, except for multiplicity, as original one from the viewpoint of the Jordan canonical form. Some examples of tridiagonalization are numerically given.

#### Global solutions to oncompressible Navier-Stokes equations with gravity

Weimin Peng, Fudan University

**Abstract:** In this talk, we consider a special class of initial data to the 3D incompressible Navier-Stokes equations with gravity. We show that, under such conditions, the incompressible Navier-Stokes equations with gravity are globally wellposed. The important features of the initial data is that the velocity fields minus gravity term are almost parallel to the corresponding vorticity fields in a very large space domain.

#### Stokes dynamics of swimming microorganisms

Kenta Ishimoto, RIMS

**Abstract:** The flow around a swimming microorganism obeys the inertialess Stokes equations with time-depending boundary conditions. Due to the linearity of the governing equations, there exists a constraint on motion of the swimmer, which is known as the scallop theorem. In this talk, the theorem and its breakdown due to small inertia will be argued. A numerical approach to such a swimmer will be also discussed.

## Regularization by projection for a backward problem of the time-fractional diffusion equation

Caixuan Ren, Fudan University

Abstract: Time-fractional diffusion equations, which frequently appear in the diffusion process, describe the continuous time random walk phenomena. In this talk we investigate a backward time-fractional diffusion equation where one wants to extract the initial temperature distribution from the observation data provided along the final time t=T. Based upon the eigenfunction expansion, an analytical solution is deduced. However, such an approach does not depend continuously on the observation data. Hence, we propose a regularization by projection method where the truncated level plays the role of the regularization parameter. Under appropriate regularity assumptions of the exact solution, a uniform error estimate with an optimal convergence rate between the reconstructed solution and the exact one is obtained both for a priori and a posteriori parameter choice strategies. Finally, numerical examples are presented to illustrate the validity and effectiveness of the proposed method.

## The uniqueness of decompositions of a (quasi)toric manifold into products of real 2 or 4 dimensional (quasi)toric manifolds

Miho Hatanaka, Osaka City University

**Abstract:** A quasitoric manifold is a closed smooth even domensional manifold with an effective smooth action of compact torus, such that it is locally equivariantly diffeomorphic to a representation space of compact torus and the orbit space is a simple convex polytope. The uniqueness of a direct decomposition of a closed smooth manifold does not hold in general up to diffeomorphism. However, it holds for decompositions of (quasi)toric manifolds into products of real 2 or 4 dimensional (quasi)toric manifolds up to diffeomorphism, which I will talk about.

#### Rational maps whose Julia sets are Cantor circles

Fei Yang, Fudan University

**Abstract:** We give a family of rational maps whose Julia sets are Cantor circles and show that every rational map whose Julia set is a Cantor set of circles must be topologically conjugate to one element of this family on their corresponding Julia sets. In particular, we give the formulas of some rational maps whose Julia sets are Cantor circles, but they are not topologically conjugate to any McMullen maps on their Julia sets. Moreover, a series of non-hyperbolic rational maps whose Julia sets are Cantor circles are constructed.

#### On Kac's scattering length formula

Jiangang Ying, Fudan University

**Abstract:** The scattering length formula was formulated and proved in special cases by Kac et al. It was discussed by a series of authors, including M. Taylor, H. Tamura, Y. Takahashi. The formula was proved by M. Takeda in symmetric case and by P. He assuming weak duality. In this talk, we shall describe the background of this formula and prove it in the general framework of right Markov processes.

#### Renormalization in complex dynamics

Mitsuhiro Shishikura, Kyoto University

Abstract: One often observes "self-similarities" within the phase space and the parameter space of chaotic dynamical systems. While the self-similarity in the phase space is easily understood via the dynamics itself, the self-similarity in the parameter space is explained by recurrent behavior of orbits and small scale bifurcations reproducing the large scale bifurcations. The renormalization is a tool to study such phenomenon. The key is to consider the return map along recurrent orbit and define it be a new dynamical system, and this construction is considered to be a meta-dynamics in the space of certain dynamical systems. In one dimensional dynamics, the renormalization has been the focus of the research for several decades in connection with the conjectures on the density of hyperbolicity and the local connectivity. In this talk, we discuss the general ideas of renormalization and recent development.

#### Water-waves and drift of particles

Hisashi Okamoto, RIMS

**Abstract:** We compute trajectories of fluid particles in a water-wave which propagates with a constant shape in a constant speed. The Stokes drift, which asserts that fluid particles are pushed forward by a wave, is proved in a new method. Numerical examples with various gravity and surface tension coefficients are presented.

#### Global solution for 2-d steady supersonic flow over a bending Wall

Yongqian Zhang, Fudan University

Abstract: We are concerned with the steady supersonic flow moving over a two dimensional bending wall. The bending wall is assumed to be a small perturbation of a convex one. Under the assumption that the total variation of the tangent of the perturbation is sufficiently small, the global approximate solutions are constructed via wave front tracking scheme, and the convergence of approximate solutions to the entropy solution is shown. Also the asymptotic behaviour of the solution is studied.

#### The existence and blow-up criterion of liquid crystals system in critical Besov space

Yihang Hao, Fudan University

**Abstract:**We build the existence of strong solution to liquid crystals system both locally and globally in critical Besov space. According to Serrin's well-known criterion, we give a general criterion for liquid crystals on Besov type. Our criterion has slight difference, as in our system we will confront the nonlinear term  $|\nabla d|^2 d$ .

#### A Monge-Ampère foliation associated with a geodesic in the space of Kähler metrics

Kenta Tottori, Tohoku University

**Abstract:** A geodesic in the space of kähler metrics was defined by Mabuchi, Semmes, Donaldson. In this talk, I want to explain the geodesic. It corresponds to a solution of Homogeneous Complex Monge-Ampère equation and the solution defines the so called Monge-Ampère foliation. In particular, the talk will focus on the foliation.

#### Sharp perturbation bounds for the total least squares problem

Pengpeng Xie, Fudan University

**Abstract:** In this talk, new perturbation analysis of the total least squares(TLS) is considered. The results are much better than the earlier results. We also apply the statistical condition estimate(SCE) to TLS. In the numerical examples, we compare our perturbation bounds and the resulting forward errors with other error estimates given in the literature. As a byproduct, We show that the condition number derived by Li and Jia (cf. [Linear Algebra Appl., 435 (2011), pp. 674–686]) is the same as that obtained by Baboulin and Gratton (cf. [SIAM J. Matrix Anal. Appl., 32 (2011), pp. 685–699]).

## Classification of semisimple symmetric spaces with properly discontinuous actions of surface groups

Takayuki Okuda, University of Tokyo

**Abstract:** We consider symmetric spaces M := G/H with connected linear simple Lie groups G. In this talk, we give a classification of such symmetric spaces for which there exist properly discontinuous actions of surface groups as isometries of M. Our method is based on the results of T. Kobayashi [Math. Ann. '89] and Y. Benoist [Ann. Math. '96] together with combinatorial techniques of nilpotent orbits.

#### p-essential normality of quasi-homogeneous Arveson's submodules

Chong Zhao, Fudan University

**Abstract:** The first author and Wang proved that each homogeneous principal submodule of the Drury-Arveson module  $H_n^2$  is essentially normal, and hence in dimensions n = 2, 3 each homogeneous submodule of  $H_n^2$  is essentially normal. For the Bergman modules  $L_a^2(B_n)$  on the unit ball, Douglas and Wang recently proved that every principal submodule is essentially normal.

We develop some new techniques to prove the essential normality of Drury-Arveson's quasihomogeneous principal submodules, by a combination of the approach of Douglas and Wang. As a consequence, we prove that each quasi-homogeneous submodule of  $H_n^2$  is essentially normal for dimensions n = 2, 3, and determine the related K-homology.

#### Super extension of Bell polynomials with applications to superymmetric equations

Engui Fan, Fudan University

**Abstract:** Bell polynomials into super version, which are found to be effective in systematically constructing super bilinear representation, bilinear Backlund transformation, Lax pair and infinite conservation laws of supersymmetric equations. We take supersymmetric KdV equation and supersymmetric sine-Gordon equation to illustrate this procedure.

#### Quasi-homomorphisms

Koji Fujiwara, Kyoto University

**Abstract:** A function f on a group G is a quasi-homomorphism if there exists a constant C such that for any two group elements g and h, |f(gh) - f(g) - f(h)| is at most C. If C = 0, then f is a homomorphism. Quasi-homomorphisms are useful to show some interesting properties of groups, and I'd like to discuss that. This will be a survey talk.

#### Complete intersections of two quadrics and Galois cohomology

Yasuhiro Ishitsuka, Kyoto University

**Abstract:** There are a classical relation between Jacobians of a hyperelliptic curve over  $\mathbb{C}$  and lines on intersection of two quadrics over  $\mathbb{C}$ . In this talk, we consider the relation over the rational number field  $\mathbb{Q}$ , and some related topics.

#### A characterization of birationality of the fourth pluricanonical map

Jinsong Xu, Fudan University

**Abstract:** The pluricanonical systems is an important way of understanding the structure of algebraic varieties. Besides the classic work of E.Bombieri on surfaces, there are a lot of progress for higher dimensional varieties in recent years. Generally, it is difficult to describe the behavior of the pluricanonical map  $\varphi_m$  when m small. In this talk, I will give a characterization of non-birationality of 4-th pluricanonical map of an algebraic threefold of general type. We show that under the assumption that volume(X) is large, the 4-th pluricanonical map is not birational if and only if X is fibred by surfaces of type (vol(S),  $p_q(S)$ ) = (1, 2).

#### A generalization of the theory of Coleman power series

Kazuto Ota, Tohoku University

**Abstract:** The classical Coleman power series theory says that every norm compatible system of local units in p-power cyclotomic fields is interpolated by a power series. There are several generalization of this theory, which play important roles in Iwasawa theory. Recently, Shinichi Kobayashi (Tohoku Univ.) found a new generalization of Coleman theory to formal groups associated to elliptic curves, and he applied it to computation of p-adic height parings. In this talk, we generalize his theory to higher dimensional formal groups.

#### Tricomi problem for a quasilinear Lavrentiev-Bitsadze equation of mixed type

Zhenguo Feng, Fudan University

**Abstract:** In this report, we consider the Tricomi problem of a quasilinear Lavrentiev-Bitsadze mixed type equation

 $(\operatorname{sgn} u_y)\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial u^2} - 1 = 0,$ 

whose coefficients depend on the first order derivative of unknown function. We prove the existence of solution to this problem by the using hodograph transformation, which can be applied to study more difficult problems for nonlinear mixed type equations arising in gas dynamics.

#### Well-posedness of the fifth order KdV equation with periodic boundary condition

Takamori Kato, Kyoto University

**Abstract:** In this talk, we consider the well-posedness of the Cauchy problem for the fifth order KdV equation in the periodic setting. This equation is completely integrable and is derived from generalizing the KdV equation in some sense. The perturbation theory does not work. Here the perturbation means the nonlinear term is regarded as an approximate linear solution. To overcome this difficulty, we focus on the rich symmetry of this equation. Actually we use the algebraic structure of the nonlinear terms and the fundamental analysis tools so that the strong nonlinear interactions are canceled and we can get the optimal result in some sense.