DAHA from scratch

Ivan Cherednik (North Carolina)

In a very rich world of Representation Theory and its applications, the sl(2) plays absolutely the key role; sl(3) is just a Lie algebra, though sl(n) are more special than other simple Lie algebras. The sl(2) has many unique features. Its affine counterpart, sl(2)-hat, changed dramatically RT and related physics; deep connections were established with the Number Theory and quite a few other brunches of mathematics.

The double affine Hecke algebra of type A_1 is a unification of sl(2) with another key object of mathematics/physics, the Heisenberg-Weyl algebra. It also extends the affine (p-adic) Hecke algebras (which gave it its name). The definition of DAHA is simple, especially for A_1, but the scope of its applications, known and expected, is impressive, from the Harmonic Analysis and Combinatorics to the theory of affine flag varieties and torus knots.

The plan is to prepare the listeners (the younger the better, no knowledge of the Lie theory is assumed) to use this relatively new tool in their research. The focus will be on the polynomial representation and its quotients (the Verlinde algebra is one of them), though we will try to establish connections with the p-adic spherical functions (likely) and the Demazure characters (possibly). The lecturer may try an active approach: questions or even inviting the volunteers(!) to the blackboard. This will be of course adjusted during the lectures.

1) DAHA and its polynomial representation.

   We will introduce DAHA algebraically, including the projective action of SL(2,Z). The polynomial representation will be introduced and used to prove the PBW theorem, one of the most important features of DAHA.

2) Nonsymmetric Macdonald polynomials.

   A systematic theory, including the technique of intertwiners, the duality theorem and its applications to the Pieri formulas, among other aspects. If time allows, the topological interpretation.

3) The Demazure limit and the p-adic limit.

   The first is $t -- > 0$; we will briefly touch the geometric aspects (affine Schubert varieties), but mainly will focus on the formulas. The second is $q -- > 0$; it will include a mini-intro into the p-adic theory.

4) Verlinde algebras; singular k, roots of unity.

   A description of the perfect representations, the remarkable quotients of the polynomial representations with very rich structures, which exist only at roots of unity or for $t = q^k$ for negative half-integers k.