

Measure-preserving actions of infinite-dimensional groups

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In this course we will study ergodic properties of actions of infinite-dimensional groups such as the infinite unitary group defined as the inductive limit of finite-dimensional unitary groups. The motivation for this study comes from the problem of harmonic analysis on infinite-dimensional groups.

For compact groups, the classical Peter-Weyl Theorem gives a decomposition of the regular representation of a compact group into irreducible ones. What would be the natural analogue of this result for the infinite unitary group?

We consider the space of infinite hermitian matrices, the projective limit of finite-dimensional spaces of hermitian matrices. The infinite unitary group acts on this space by conjugation, and ergodic invariant probability measures of this action admit an explicit classification due to Pickrell and to which a different approach was proposed by Olshanski and Vershik. In the course we will study the Pickrell-Olshanski-Vershik classification, the problem of ergodic decomposition for actions of infinite-dimensional groups and its relation to the problem of harmonic analysis on our groups.

1. Introduction to Harmonic Analysis on Infinite-Dimensional Groups.
2. Ergodic Measures: the Pickrell Classification, the Olshanski-Vershik Approach.
3. Finiteness of Ergodic Unitarily-Invariant Measures on Spaces of Infinite Matrices.
4. Determinantal Point Processes.