POLYNOMIAL ROOT-FINDING METHODS WHOSE BASINS OF ATTRACTION APPROXIMATE VORONOI DIAGRAM

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Abstract. Given a complex polynomial $p(z)$ with at least three distinct roots, we first prove no rational iteration function exists where the basin of attraction of a root coincides with its Voronoi cell. In spite of this negative result, we prove the Voronoi diagram of the roots can be well approximated through a high order sequence of iteration functions, the *Basic Family*, $B_m(z)$, $m \geq 2$. Let θ be a simple root of $p(z)$, $V(\theta)$ its Voronoi cell, and $A_m(\theta)$ its basin of attraction with respect to $B_m(z)$. We prove that given any closed subset C of $V(\theta)$, including any homothetic copy of $V(\theta)$, there exists m_0 such that for all $m \geq m_0$, C is also a subset of $A_m(\theta)$. This implies when all roots of $p(z)$ are simple, the basins of attraction of $B_m(z)$ uniformly approximates the Voronoi diagram of the roots to within any prescribed tolerance. Equivalently, the Julia set of $B_m(z)$, and hence the chaotic behavior of its iterations, will uniformly lie to within prescribed strip neighborhood of the boundary of Voronoi diagram. In a sense this is the strongest property a rational iteration function can exhibit for polynomials. Next, we use the results to define and prove an infinite layering within each Voronoi cell of a given set of points, whether known implicitly as roots of a polynomial equation, or explicitly via their coordinates. We discuss potential application of our layering in computational geometry.

Key words. Complex polynomials; Voronoi diagrams; zeros; Newton's method; iteration functions; fractal; Julia set, dynamical systems; computational geometry