"Advanced numerical methods for dynamical systems"

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Lecture series at Kyoto University

Tuesday, March 10

10:30 - 12:00
Introduction to numerical continuation of solutions of differential equations (given by BK; this first lecture will be done on a blackboard)

This lecture provides an introduction to the concept of numerical path following of solutions of autonomous differential equations. The key idea is to define the object under consideration as the solution of a well-posed root finding problem (with isolated roots). This is straightforward for equilibria and their bifurcations. To continue periodic orbits, and orbit segments of interest more generally, they need to be represented and solvedfor as solutions of two-point boundary value problems.

Reference:
B. Krauskopf, H.M. Osinga and J. Galan-Vioque (Eds.), Numerical Continuation Methods for Dynamical Systems: Path following and boundary value problems, Springer-Verlag (2007)

2:00 - 3:00
Computing (un)stable manifolds of (global) Poincare maps (given by HMO)

In this lecture we discuss the problem of computing 1D global stable and unstable manifolds of a fixed point of the return map of a vector field, defined on a given Poincare section. The return map is locally a diffeomorphism, but, in general, this property does not hold on the entire Poincare section. More specifically, there is a locus of codimension one in the Poincare section where the flow is tangent to the section. We present a numerical method, based on the continuation of orbit segments, that is able to compute one-dimensional global manifolds of the Poincare map, even when they cross the tangency locus. Furthermore, we show that our method also works when the flow under consideration features vastly differing contraction rates, as is typical for systems with different time scales.

Reference:
J.P. England, B. Krauskopf and H.M. Osinga, Computing one-dimensional global manifolds of Poincare maps by continuation, SIAM Journal on Applied Dynamical Systems 4(4) (2005) 1008-1041

3:15 - 4:15
The computation of slow manifolds and canard orbits in slow-fast vector fields (given by BK)

We consider a vector field with two time scales; specifically with two slow and one fast variable. It is well known that such systems present numerical challenges. In particular, the initial value problem becomes ill posed. An important object is the critical manifold, which is given as the nullcline of the fast flow. In our context of one fast and two slow variables, the critical manifold is a smooth two-dimenisonal surface with attracting and repelling sheets that meet generically along fold curves with respect to the fast variable. When the ratio of time scales is nonzero, the sheets of the critical manifold perturb to smooth surfaces called slow manifolds. The attracting and repelling slow manifolds may intersect near special points of the fold curve. Such intersection curves are known as canard orbits, and they are closely associated with the small-amplitude component of mixed-mode oscillations. We present a method for computing slow manifolds and canard orbits, which also allows us to continue the latter in system parameters.

References:
[1] M. Desroches, B. Krauskopf and H.M. Osinga, The geometry of slow manifolds near a folded node, SIAM Journal on Applied Dynamical Systems 7(4) (2008) 1131-1162
[2] M. Desroches, B. Krauskopf and H.M. Osinga, Mixed-mode oscillations and slow manifolds in the self-coupled FitzHugh-Nagumo system, CHAOS 18(1) (2008) 015107

4:30 - 5:30
Computing global manifolds of vector fields as a sequence of geodesic level sets (given by HMO)

We present a method for the computation of global stable and unstable manifolds of equilibria and periodic orbits of vector fields. More specifically, the respective global manifold is grown step by step by adding rings of new mesh points that lie to good approximation on geodesic level sets of the unknown manifold. Each new mesh point is found by solving a two-point boundary value problem. The computed part of the global manifold is shown to satisfy rigorous error bounds. The method is demonstrated with several example vector field, including the Lorenz system.

References:
[1] B. Krauskopf and H.M. Osinga, Computing geodesic level sets on global (un)stable manifolds of vector fields, SIAM Journal on Applied Dynamical Systems 2 (4) (2003) 546-569
[2] B. Krauskopf , H.M. Osinga, E.J. Doedel, M.E. Henderson, J. Guckenheimer, A. Vladimirsky, M. Dellnitz and O. Junge, A survey of methods for computing (un)stable manifolds of vector fields, Int. J. Bifurcation and Chaos 15(3) (2005) 763-791

Wednesday, March 11

10:30 - 11:30
The symbolic dynamics of heteroclinic orbits of the Lorenz system (given by BK)

The Lorenz manifold is the 2D stable manifold of the origin of the famous Lorenz system. We consider here its intersections with the 2D unstable manifolds of the secondary equilibria or periodic orbits of saddle type (also known as the template of the Lorenz system). These intersection curves are structurally stable heteroclinic connections from the origin to the secondary equilibria. We compute 512 of these heteroclinic orbits and continue them in the Reynolds number parameter of the Lorenz system. We show how the symbolic dynamics of these heteroclinic orbits is associated with the symbolic dynamics of codimension-one homoclinic orbits of the origin, at which they originate and terminate.

Reference:
E.J. Doedel, B. Krauskopf and H.M. Osinga, Global bifurcations of the Lorenz manifold, Nonlinearity 19(12) (2006) 2947-2972

11:45 - 12:45
Visualization techniques for global manifolds: from the equations to the computer screen to steel (given by HMO)

Global manifolds of dynamical systems may be very complicated geometrical objects. Therefore, it is important to visualise them appropriately after they have been computed. We demonstrate that the computation and representation of a 2D global manifold as a collection of geodesic level sets can be used in different ways to bring out its geometry by computer illustrations --- and even as a sculpture in steel.

References:
[1] H.M. Osinga and B. Krauskopf, Visualizing chaos in the Lorenz system, Computers and Graphics 26(5) (2002) 815-823
[2] H.M. Osinga and B. Krauskopf, Visualizing curvature on the Lorenz manifold, Journal of Mathematics and the Arts 1(2) (2007) 113-123
[3] H.M. Osinga, B. Krauskopf and B. Storch, The sculpture Manifold: a band from a surface, a surface from a band, in Reza Sarhangi and Carlo Sequin (Eds.) Proceedings of Bridges Leeuwarden: Mathematical Connections in Art, Music, and Science, Leeuwarden, pp. 9-14, 2008.