

# Russia-Japan School of Young Mathematicians

Date: 14 Jan. – 2 Feb.

Place: Room 305 of Building No.3, Department of Mathematics, Kyoto University.

We have lectures given by professors as the following table. The other weekdays, we have tea-time and talks of participants from 14:00 to 17:00. We prepare a seminar room for discussion at Room 308 of Building No.3. Note that this room is not available during 11:00 – 13:00.

## Lectures

16 Jan. (Fri.) at Conference room (Room 127) of Building No.3

11:00 – 12:00 Tomoyuki Arakawa

Chiral differential operators and affine localization of  $\mathfrak{g}$ -modules

13:30 – Hiraku Nakajima

Problems on quiver varieties and quantum enveloping algebras

20 Jan. (Tue.)

16:15 – 17:15 Vladimir Belavin

Super minimal Liouville gravity

22 Jan. (Thu.)

14:00 – 15:00 Dmitry Kaledin

Symplectic resolutions in geometric representation theory

15:15 – 16:15 Boris Feigin

Action of one version of quantum  $\mathfrak{gl}_\infty$  in the equivariant K-theory of Hilbert scheme

16:15 – 17:15 Alexander Belavin

Two-dimensional quantum gravity

27 Jan. (Tue.)

14:00 – 15:00 Syu Kato

Anti-spherical discrete series of affine Hecke algebras and its applications

30 Jan. (Fri.) at Conference room (Room 127) of Building No.3

11:00 – 12:00 Takeshi Suzuki

Cylindrical combinatorics and the representation theory of Cherednik algebras

13:30 – Hiraku Nakajima

Duality among four theories

## Time Table of Tea-time and Talks

15 Jan. (Thu.)

10:00 – Vadim Gorin

Probability models connected with 3-dimensional Young diagrams

16 Jan. (Fri.) Lectures at Conferenc room (Room 127) of Building No.3

11:00 – 12:00 Tomoyuki Arakawa

Chiral differential operators and affine localization of  $\mathfrak{g}$ -modules

13:30 – Hiraku Nakajima

Problems on quiver varieties and quantum enveloping algebras

19 Jan. (Mon.)

15:00 – 15:50 Vadim Gorin

Stochastic dynamics on the space of stepped surfaces

15:50 – 16:00 break

16:00 – 17:00 Shunsuke Tsuchioka

Catalan numbers and level 2 weight structures of  $A_{p-1}^{(1)}$

20 Jan. (Tue.) at Room 110 of Building No.3

15:00 – 16:00 Evgeny Smirnov

Schubert decomposition for double Grassmannians

16:00 – 16:15 break

16:15 – 17:15 Vladimir Belavin

Super minimal Liouville gravity

21 Jan. (Wed.)

14:00 – 15:00 Alexander Gaifullin

Combinatorial realization of cycles

15:00 – 15:30 Motoo Tange

Homology 3-spheres yielding lens spaces by Dehn surgery

15:30 – 15:45 break

15:45 – 16:30 Maxim Karev

Epimorphisms of groups of knots and generalization of Alexander polynomial

22 Jan. (Thu.) Lectures

14:00 – 15:00 Dmitry Kaledin

Symplectic resolutions in geometric representation theory

15:00 – 15:15 break

15:15 – 16:15 Boris Feigin

Action of one version of quantum  $\mathfrak{gl}_\infty$  in the equivariant K-theory of Hilbert scheme

16:15 – 17:15 Alexander Belavin

Two-dimensional quantum gravity

23 Jan. (Fri.)

14:00 – 15:00 Sergey Ryobakov

Abelian varieties over finite fields

15:00 – 15:30 Hiroyuki Minamoto

A noncommutative algebro-geometric characterization of representation type of a quiver

15:30 – 15:45 break

15:45 – 16:30 Alexey Elagin

Equivariant derived categories and descent theory

16:30 – 17:20 Vadim Gorin

Limit shapes of 2-dimensional Young diagrams

26 Jan. (Mon.) at Room 110 of Building No.3

14:00 – 15:00 Kento Nakada

Colored hook formula for a generalized Young diagram

15:00 – 15:40 Syunji Moriya

A generalization of Sullivan-de Rham equivalence to spaces with finite fundamental group

15:40 – 16:00 break

16:00 – 17:00 Alexander Gaifullin

Combinatorial formulae for the Pontryagin classes of triangulated manifolds

27 Jan. (Tue.)

14:00 – 15:00 Syu Kato

Anti-spherical discrete series of affine Hecke algebras and its applications

15:00 – 15:15 break

15:15 – 16:15 Yoshiyuki Kimura

Affine quiver and the crystal  $B(\infty)$

16:15 – 17:00 Takuya Shinkado

On the center of double affine Hecke algebra

28 Jan. (Wed.)

14:00 – 15:00 Yukihiro Uchida

Division polynomials of hyperelliptic Jacobians

15:00 – 16:00 Toshiro Kuwabara

A rational Cherednik algebra as a noncommutative deformation of a quiver variety

16:00 – 16:15 break

16:15 – 17:15 Tetsuya Abe

Alternating knots in Gordian space of knots

29 Jan. (Thu.)

14:00 – 15:00 Azat Gaynutdinov

Quantum groups, Logarithmic CFT, and XXZ models

15:00 – 15:10 break

15:10 – 15:40 Nobuharu Sawada

A structure of the cyclotomic  $q$ -Schur algebras

15:40 – 16:40 Daisuke Yamakawa

Regular connections in dimension one and star-shaped quivers

16:40 – 17:40 Ryosuke Kodera

A generalization of adjoint crystals for the quantized affine algebras of type  $A_n^{(1)}$ ,  $C_n^{(1)}$  and  $D_{n+1}^{(2)}$

30 Jan. (Fri.) Lectures at Conference room (Room 127) of Building No.3

11:00 – 12:00 Takeshi Suzuki

Cylindrical combinatorics and the representation theory of Cherednik algebras

13:30 – Hiraku Nakajima

Duality among four theories

## Abstracts

Speaker: Vadim Gorin

Title: Probability models connected with 3-dimensional Young diagrams

I will review the main results in the study of random stepped surfaces in  $R^3$  (equivalently, random 3-dimensional Young diagrams or lozenge tilings of regions of the plane) in the last ten years including the most recent results. I will speak about limit shapes, structure of the fluctuations and limit measures that arise in various models. Although these models have been intensively studied, there are still some conjectures that I'm going to mention, and which still remain unproved.

Title: Stochastic dynamics on the space of stepped surfaces

I will describe two Markov chains: One of them provides an efficient algorithm for sampling random stepped surfaces, while another one gives a new stochastic surface growth model that unites 3 simple processes studied by lots of authors in the different context. Also I will try to explain the connection between stepped surfaces and representation theory.

Speaker: Sergey Rybakov

Title: Abelian varieties over finite fields

I will speak on some basic facts and notions: endomorphism algebras, Frobenius action on Tate module and the Honda-Tate theorem, classification of isogeny classes using Weil numbers and zeta functions, groups of rational points.

Speaker: Evgeny Smirnov

Title: Schubert decomposition for double Grassmannians

Classical Schubert calculus deals with orbits of a Borel subgroup  $B \subset GL(V)$  acting on a Grassmann variety  $Gr(k, V)$  of  $k$ -planes in a finite-dimensional vector space  $V$ . These orbits (Schubert cells) and their closures (Schubert varieties) are very well studied both from the combinatorial and the geometric points of view.

One can go one step farther, considering the direct product of two Grassmannians  $Gr(k, V) \times Gr(l, V)$  and the Borel subgroup  $B \subset GL(V)$  acting diagonally on this variety. In this case, the number of orbits still remains finite, but their combinatorics and geometry of their closures become much more involved. It would be challenging to extend the whole body of the Schubert calculus to this situation.

I will explain how to index the  $B$ -orbit closures in  $Gr(k, V) \times Gr(l, V)$  combinatorially, describe the inclusion relations between them, and construct their desingularizations, which are analogous to Bott-Samelson desingularizations for ordinary Schubert varieties. If time allows, I will also try to discuss the relations of this situation with geometry of quiver representations; these relations were recently found by Bobinski and Zwara.

Speaker: Yoshiyuki Kimura

Title: Affine quiver and the crystal  $B(\infty)$

We study a relationship between the two geometric construction of the crystal  $B(\infty)$  in (untwisted simply-laced) affine cases. One is the Lusztig's canonical base (as simple perverse sheaves) and another is the Lagrangian construction by using Lusztig's quiver varieties due to Kashiwara-Saito. More precisely, we study some estimates of the micro-support (or characteristic variety) of simple perverse sheaves (corresponding the canonical base the canonical basis of affine quivers described by Lusztig).

Speaker: Ryosuke Kodera

Title: A generalization of adjoint crystals for the quantized affine algebras of type  $A_n^{(1)}$ ,  $C_n^{(1)}$  and  $D_{n+1}^{(2)}$

The adjoint crystal is a certain combinatorial crystal, which is defined by Benkart-Frenkel-Kang-Lee. They gave a uniform construction of them for all quantized affine algebras and proved that they are level one perfect. On the other hand, we can construct a crystal  $B_l$  for every nonnegative integer  $l$  by using Kirillov-Reshetikhin modules. We see that this family of crystals is a natural generalization of adjoint crystals ( $B_1$  coincides with the adjoint crystal). In this talk, I explain the structure of these crystals for the quantized affine algebras of type  $A_n^{(1)}$ ,  $C_n^{(1)}$  and  $D_{n+1}^{(2)}$ .

Speaker: Syunji Moriya

Title: a generalization of Sullivan-de Rham equivalence to spaces with finite fundamental group

I talk about a generalization of Sullivan's de Rham homotopy theory to non-simply connected spaces, which will be suitable for geometric application. the formulation is such that real homotopy type of a manifold should be the dg-category of flat bundles on the manifold much the same as real homotopy type of a simply connected manifold is the dg-algebra of differential forms with constant coefficients in original Sullivan's theory. I explain an equivalence (a Sullivan-de Rham type equivalence) between the rational homotopy category of spaces whose fundamental groups are finite, whose higher homotopy groups have finite rank and the homotopy category of dg-categories having certain additional structures.

Speaker: Kento Nakada

Title: Colored hook formula for a generalized Young diagram

As is well-known, the number of standard Young tableaux is given by hook length formula. In this talk, we give a colored hook formula, as a generalization of this result, for a generalized Young diagram in the sense of D. Peterson and R. A. Proctor. The colored hook formula is an identity in multivariable rational functions. Furthermore, this is new even for a classical Young diagram.

Speaker: Nobuharu Sawada

Title: A structure of the cyclotomic q-Schur algebras

The cyclotomic q-Schur algebra was introduced by Dipper-James-Mathas to study the representation theory for the Ariki-Koike algebra (which associate with the complex reflection group  $G(r, l, n)$ ). In my talk, we introduce some "good" subalgebra of the cyclotomic q-Schur algebra and explain some properties of that subalgebra and the relationship between them. In particular, we discuss around the topics on their decomposition numbers.

Speaker: Motoo Tange

Title: Homology 3-spheres yielding lens spaces by Dehn surgery

J.Berge conjectured that doubly primitive knots in  $S^3$  are all knots yielding lens spaces. We show that doubly primitive knots in some plumbed homology spheres may yield lens spaces.

Speaker: Shunsuke Tsuchioka

Title: Catalan numbers and level 2 weight structures of  $A_{p-1}^{(1)}$

Motivated by a connection between representation theory of the degenerate affine Hecke algebra of type A and Lie theory associated with  $A_{p-1}^{(1)}$ , we determine the complete set of representatives of the orbits for the Weyl group action on the set of weights of level 2 integrable highest weight representations of  $\widehat{\mathfrak{sl}}_p$ . Applying a crystal technique, we show that Catalan numbers appear in their weight multiplicities.

Speaker: Yukihiro Uchida

Title: Division polynomials of hyperelliptic Jacobians

In the theory of elliptic curves, division polynomials are important for the study of the structure of the torsion subgroup. Y. Onishi recently generalized division polynomials to the case of hyperelliptic curves (and more general curves) and gave determinantal expressions of them. In this talk, we will generalize division polynomials to the case of hyperelliptic Jacobians over the complex numbers. We will also give determinantal expressions and some properties of division polynomials.