Symbolic Methods For Differential Geometry and its Applications

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Computer algebra systems such as Maple and Mathematica play an every increasingly important role in mathematics teaching and research across a broad range of disciplines. In this series of lectures, I shall discuss the use of Maple in the area of differential geometry and its applications to differential equations, the calculus of variations and mathematical physics. The format for each lecture will be the same - we shall review the relevant mathematical concepts and then illustrate these concepts using the *DifferentialGeometry* software package. Some details of various algorithms will be presented and on-going developments and open problems will be discussed. The software demonstrations will be made available to all conference participants. (Participants may wish to bring their laptops (with Maple - Release 11 or higher) but this is not required.)

I. An Introduction to Differential Geometry with Maple.

1. A brief introduction to the computer algebra system Maple. 2. An overview of the *DifferentialGeometry* software project. 3. Vector Fields, Flows and Brackets. 4. Invariants. 5. Differential Forms and Tensors. 6. Killing Vectors. 7. Symmetries of Differential Equations.

II. An Introduction to Lie Algebras and Lie Groups with Maple.

1. The Structure theory for Lie algebras – Historical Remarks. 2. The *DifferentialGeometry* database of Lie algebras. 3. The Fundamental Theorems of Lie. 4. Classification Problems. 5. Application to the Equivalence Problem in General Relativity

III. Applications to Differential Equations.

1. Distributions and Pfaffian Systems. 2. Integral Manifolds. 3. External and Internal Geometries. 4. Using Lie groups to solve differential equations I. 5. Using Lie groups to solve differential equations II.

IV. Applications to the Calculus of Variations.

1. Euler-Lagrange Operators. 2. Jet Spaces. 2. The Inverse problem of the Calculus of Variations. 3. The Principle of Symmetric Criticality.